Constraining penguin contributions and the CKM angle γ through $B_d \to \pi^+\pi^-$

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Abstract. The decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ provide an interesting strategy to extract the CKM angle γ at "second-generation" B-physics experiments of the LHC era. A variant for "first-generation" experiments can be obtained, if $B_s \to K^+K^-$ is replaced by $B_d \to \pi^\mp K^\pm$. We show that the most recent experimental results for the CP-averaged $B_d \to \pi^+\pi^-$ and $B_d \to \pi^\mp K^\pm$ branching ratios imply a rather restricted range for the corresponding penguin parameters, and upper bounds on the direct CP asymmetries $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)$. Moreover, we point out that interesting constraints on γ can be obtained from the CP-averaged $B_d \to \pi^+\pi^-$ and $B_d \to \pi^\mp K^\pm$ branching ratios, if in addition mixing-induced CP violation in the former decay is measured, and the B_d^0–$\overline{B_d^0}$ mixing phase is fixed through $B_d \to J/\psi K_{\rm S}$. An extraction of γ becomes possible, if furthermore direct CP violation in $B_d \to \pi^+\pi^-$ or $B_d \to \pi^\mp K^\pm$ is observed.

1 Introduction

Among the central targets of the B-factories is a measurement of the time-dependent CP asymmetry of the decay $B_d \to \pi^+\pi^-$ [1], which can be expressed as follows:

$$a_{\rm CP}(B_d(t) \to \pi^+\pi^-)$$
$$\equiv \frac{{\rm BR}(B_d^0(t) \to \pi^+\pi^-) - {\rm BR}(\overline{B_d^0}(t) \to \pi^+\pi^-)}{{\rm BR}(B_d^0(t) \to \pi^+\pi^-) + {\rm BR}(\overline{B_d^0}(t) \to \pi^+\pi^-)}$$
$$= \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) \cos(\Delta M_d t)$$
$$+ \mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-) \sin(\Delta M_d t). \tag{1}$$

Here $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ are due to "direct" and "mixing-induced" CP violation, respectively. In the summer of 1999, the CLEO collaboration reported the first observation of the long-awaited $B_d \to \pi^+\pi^-$ transition, with the following CP-averaged branching ratio [2]:

$${\rm BR}(B_d \to \pi^+\pi^-)$$
$$\equiv \frac{1}{2}\left[{\rm BR}(B_d^0 \to \pi^+\pi^-) + {\rm BR}(\overline{B_d^0} \to \pi^+\pi^-)\right]$$
$$= \left(4.3^{+1.6}_{-1.4} \pm 0.5\right) \times 10^{-6}. \tag{2}$$

This channel usually appears in the literature as a tool to determine the angle $\alpha = 180^\circ - \beta - \gamma$ of the unitarity triangle [3] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [4]. However, penguin topologies are expected to affect this determination severely. Although there are several strategies on the market to control these penguin uncertainties [1], they are usually very challenging from an experimental point of view. Constraints on α from $B_d \to \pi^+\pi^-$ were considered in [5]–[7].

In a recent paper [8], a strategy was proposed, where $B_d \to \pi^+\pi^-$ is combined with its U-spin counterpart $B_s \to K^+K^-$ [9] to extract $\phi_d = 2\beta$ and γ. If the phase-convention independent quantity ϕ_d, which is related to the B_d^0–$\overline{B_d^0}$ mixing phase and can be determined straightforwardly with the help of the "gold-plated" mode $B_d \to J/\psi K_{\rm S}$ [10], is used as an input, the U-spin arguments in the extraction of γ can be minimized. This approach, which relies only on the U-spin flavour symmetry and is *not* affected by any final-state-interaction effects [11], is very promising for "second-generation" B-physics experiments at hadron machines, such as LHCb or BTeV [12]. There is a variant of this strategy for the asymmetric e^+e^- B-factories operating at the $\Upsilon(4S)$ resonance (BaBar and BELLE), where B_s decays cannot be explored, if $B_s \to K^+K^-$ is replaced by $B_d \to \pi^\mp K^\pm$, and a certain dynamical assumption concerning "exchange" and "penguin annihilation" topologies is made. Although $B_s \to K^+K^-$ should be accessible at HERA-B and Run II of the Tevatron, a measurement of $B_d \to \pi^\mp K^\pm$ may be easier for these "first-generation" hadronic B experiments. At HERA-B, for instance, one expects to collect 260 and 35 decay events per year of $B_d \to \pi^\mp K^\pm$ and $B_s \to K^+K^-$, respectively [13]. The present result for the CP-averaged $B_d \to \pi^\mp K^\pm$ branching ratio from the CLEO collaboration is as follows [2]:

$${\rm BR}(B_d \to \pi^\mp K^\pm)$$
$$\equiv \frac{1}{2}\left[{\rm BR}(B_d^0 \to \pi^- K^+) + {\rm BR}(\overline{B_d^0} \to \pi^+ K^-)\right]$$
$$= \left(17.2^{+2.5}_{-2.4} \pm 1.2\right) \times 10^{-6}; \tag{3}$$

a first result for the corresponding direct CP asymmetry is also available [14]:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)$$

$$\equiv \frac{{\rm BR}(B_d^0 \to \pi^- K^+) - {\rm BR}(\overline{B_d^0} \to \pi^+ K^-)}{{\rm BR}(B_d^0 \to \pi^- K^+) + {\rm BR}(\overline{B_d^0} \to \pi^+ K^-)}$$

$$= 0.04 \pm 0.16 \,. \tag{4}$$

In this paper, we point out that the CLEO results (2) and (3) imply – among other things – a rather restricted range for the ratio of the "penguin" to "tree" contributions of the decay $B_d \to \pi^+\pi^-$, and upper bounds on the direct CP asymmetries $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)$. If in addition mixing-induced CP violation in $B_d \to \pi^+\pi^-$ is measured and ϕ_d is fixed through $B_d \to J/\psi K_{\rm S}$, we may obtain moreover interesting constraints on γ. An extraction of this angle becomes possible, if direct CP violation in $B_d \to \pi^+\pi^-$ or $B_d \to \pi^\mp K^\pm$ is observed.

The outline of this paper is as follows: in Sect. 2, we have a brief look at the general structure of the relevant decay amplitudes and observables. The constraints on the penguin parameters and the direct CP asymmetries are discussed in Sect. 3, whereas the bounds on γ are the subject of Sect. 4. Finally, the conclusions and an outlook are given in Sect. 5.

2 Decay amplitudes and observables

The transition amplitude of the $\bar{b} \to \bar{d}$ decay $B_d^0 \to \pi^+\pi^-$ can be written as follows [15]:

$$A(B_d^0 \to \pi^+\pi^-) = \lambda_u^{(d)} \left(A_{\rm cc}^u + A_{\rm pen}^u \right) + \lambda_c^{(d)} A_{\rm pen}^c$$

$$+ \lambda_t^{(d)} A_{\rm pen}^t \,, \tag{5}$$

where $A_{\rm cc}^u$ is due to "current–current" contributions, the amplitudes $A_{\rm pen}^j$ describe "penguin" topologies with internal j quarks ($j \in \{u, c, t\}$), and the

$$\lambda_j^{(d)} \equiv V_{jd} V_{jb}^* \tag{6}$$

are the usual CKM factors. Making use of the unitarity of the CKM matrix and applying the Wolfenstein parametrization [16], generalized to include non-leading terms in λ [17], yields [8]

$$A(B_d^0 \to \pi^+\pi^-) = e^{i\gamma} \left(1 - \frac{\lambda^2}{2} \right) \mathcal{C} \left[1 - d\, e^{i\theta} e^{-i\gamma} \right], \tag{7}$$

where

$$\mathcal{C} \equiv \lambda^3 A\, R_b \left(A_{\rm cc}^u + A_{\rm pen}^{ut} \right), \tag{8}$$

with $A_{\rm pen}^{ut} \equiv A_{\rm pen}^u - A_{\rm pen}^t$, and

$$d\, e^{i\theta} \equiv \frac{1}{(1 - \lambda^2/2) R_b} \left(\frac{A_{\rm pen}^{ct}}{A_{\rm cc}^u + A_{\rm pen}^{ut}} \right). \tag{9}$$

The quantity $A_{\rm pen}^{ct}$ is defined in analogy to $A_{\rm pen}^{ut}$, and the CKM factors are given as usual by $\lambda \equiv |V_{us}| = 0.22$, $A \equiv |V_{cb}|/\lambda^2 = 0.81 \pm 0.06$ and $R_b \equiv |V_{ub}/(\lambda V_{cb})| = 0.41 \pm 0.07$. The "penguin parameter" $d\, e^{i\theta}$, which measures – sloppily speaking – the ratio of the $B_d \to \pi^+\pi^-$ "penguin" to "tree" contributions, will play a central role in this paper.

Using the Standard-Model parametrization (7), we obtain [8]

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = - \left[\frac{2\, d \sin\theta \sin\gamma}{1 - 2\, d \cos\theta \cos\gamma + d^2} \right] \tag{10}$$

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-) = \tag{11}$$

$$+ \left[\frac{\sin(\phi_d + 2\gamma) - 2\, d \cos\theta \sin(\phi_d + \gamma) + d^2 \sin\phi_d}{1 - 2\, d \cos\theta \cos\gamma + d^2} \right],$$

where $\phi_d = 2\beta$ can be determined with the help of the "gold-plated" mode $B_d \to J/\psi K_{\rm S}$ through

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to J/\psi K_{\rm S}) = -\sin\phi_d. \tag{12}$$

Strictly speaking, mixing-induced CP violation in $B_d \to J/\psi K_{\rm S}$ probes $\phi_d + \phi_K$, where ϕ_K is related to the weak K^0–$\overline{K^0}$ mixing phase and is negligibly small in the Standard Model. Due to the small value of the CP-violating parameter ε_K of the neutral kaon system, ϕ_K can only be affected by very contrived models of new physics [18].

In the case of $B_s \to K^+K^-$, we have [8]

$$A(B_s^0 \to K^+K^-)$$

$$= e^{i\gamma} \lambda\, \mathcal{C}' \left[1 + \left(\frac{1 - \lambda^2}{\lambda^2} \right) d'\, e^{i\theta'} e^{-i\gamma} \right], \tag{13}$$

where

$$\mathcal{C}' \equiv \lambda^3 A\, R_b \left(A_{\rm cc}^{u'} + A_{\rm pen}^{ut'} \right) \tag{14}$$

and

$$d'\, e^{i\theta'} \equiv \frac{1}{(1 - \lambda^2/2) R_b} \left(\frac{A_{\rm pen}^{ct'}}{A_{\rm cc}^{u'} + A_{\rm pen}^{ut'}} \right) \tag{15}$$

correspond to (8) and (9), respectively. The primes remind us that we are dealing with a $\bar{b} \to \bar{s}$ transition. It should be emphasized that (7) and (13) are completely general parametrizations of the $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decay amplitudes within the Standard Model, relying only on the unitarity of the CKM matrix. In particular, these expressions take into account also final-state-interaction effects, which received a lot of attention in the recent literature [11].

Since the decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other by interchanging all down and strange quarks, the U-spin flavour symmetry of strong interactions implies

$$d\, e^{i\theta} = d'\, e^{i\theta'}. \tag{16}$$

Interestingly, this relation is not affected by U-spin-breaking corrections within a certain model-dependent approach (a modernized version of the "Bander–Silverman–Soni mechanism" [19]), making use – among other things – of the "factorization" hypothesis to estimate the relevant hadronic matrix elements [8]. It would

be interesting to investigate the U-spin-breaking corrections to (16) also within the "QCD factorization" approach, which was recently proposed in [20]. In this paper, it was argued that there is a heavy-quark expansion for non-leptonic B-decays into two light mesons, and that non-factorizable corrections, as well as final-state-interaction processes, are suppressed by $\Lambda_{\rm QCD}/m_b$. We shall come back to this approach in Sect. 3, where a comparison of its prediction for the penguin parameter $d\,e^{i\theta}$ is made with the constraints that are implied by the CLEO results (2) and (3).

For the following considerations, it is useful to introduce the observable

$$H \equiv \frac{1}{\epsilon} \left| \frac{\mathcal{C}'}{\mathcal{C}} \right|^2 \left[\frac{M_{B_d}}{M_{B_s}} \frac{\Phi(M_K/M_{B_s}, M_K/M_{B_s})}{\Phi(M_\pi/M_{B_d}, M_\pi/M_{B_d})} \frac{\tau_{B_s}}{\tau_{B_d}} \right]$$
$$\times \left[\frac{{\rm BR}(B_d \to \pi^+\pi^-)}{{\rm BR}(B_s \to K^+K^-)} \right], \qquad (17)$$

where

$$\epsilon \equiv \frac{\lambda^2}{1 - \lambda^2}, \qquad (18)$$

and

$$\Phi(x,y) \equiv \sqrt{[1 - (x+y)^2][1 - (x-y)^2]} \qquad (19)$$

denotes the usual two-body phase-space function. The CP-averaged branching ratio ${\rm BR}(B_s \to K^+K^-)$ can be extracted from the corresponding "untagged" rate [8], where no rapid oscillatory $\Delta M_s t$ terms are present [21]. In the strict U-spin limit, we have $|\mathcal{C}'| = |\mathcal{C}|$. Corrections to this relation can be calculated within the "factorization" approximation, yielding

$$\left| \frac{\mathcal{C}'}{\mathcal{C}} \right|_{\rm fact} = \frac{f_K}{f_\pi} \frac{F_{B_sK}(M_K^2; 0^+)}{F_{B_d\pi}(M_\pi^2; 0^+)} \left(\frac{M_{B_s}^2 - M_K^2}{M_{B_d}^2 - M_\pi^2} \right), \qquad (20)$$

where f_K and f_π denote the kaon and pion decay constants, and the form factors $F_{B_sK}(M_K^2; 0^+)$ and $F_{B_d\pi}(M_\pi^2; 0^+)$ parametrize the hadronic quark-current matrix elements $\langle K^-|(\bar{b}u)_{\rm V-A}|B_s^0 \rangle$ and $\langle \pi^-|(\bar{b}u)_{\rm V-A}|B_d^0 \rangle$, respectively [22]. If we employ (7) and (13), we obtain the expression

$$H = \frac{1 - 2\,d\cos\theta\cos\gamma + d^2}{\epsilon^2 + 2\,\epsilon\,d'\cos\theta'\cos\gamma + d'^2}, \qquad (21)$$

which will play a key role in the following sections. Let us also note that there is an interesting relation between H and the corresponding direct CP asymmetries [8]:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$$
$$= -\epsilon H \left(\frac{d'\sin\theta'}{d\sin\theta} \right) \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-). \qquad (22)$$

Since the decays $B_s \to K^+K^-$ and $B_d \to \pi^\mp K^\pm$ differ only in their spectator quarks, we have

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) \approx \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm) \qquad (23)$$

$${\rm BR}(B_s \to K^+K^-) \approx {\rm BR}(B_d \to \pi^\mp K^\pm) \frac{\tau_{B_s}}{\tau_{B_d}}, \qquad (24)$$

and obtain

$$H \approx \frac{1}{\epsilon} \left(\frac{f_K}{f_\pi} \right)^2 \left[\frac{{\rm BR}(B_d \to \pi^+\pi^-)}{{\rm BR}(B_d \to \pi^\mp K^\pm)} \right] = 7.4 \pm 3.0. \quad (25)$$

Here we have also taken into account the CLEO results (2) and (3), and have added the experimental errors in quadrature. The advantage of (25) is that it allows the determination of H without a measurement of the decay $B_s \to K^+K^-$. However, it should be kept in mind that this relation relies not only on $SU(3)$ flavour-symmetry arguments, but also on a certain dynamical assumption. The point is that $B_s \to K^+K^-$ receives also contributions from "exchange" and "penguin annihilation" topologies, which are absent in $B_d \to \pi^\mp K^\pm$. It is usually assumed that these contributions play a minor role [23]. However, they may be enhanced through certain rescattering effects [11]. Although these topologies do *not* lead to any problems in the strategies discussed below if H is fixed through a measurement of $B_s \to K^+K^-$ – even if they should turn out to be sizeable – they may affect (23)–(25). The importance of the "exchange" and "penguin annihilation" topologies contributing to $B_s \to K^+K^-$ can be probed – in addition to (23) and (24) – with the help of the decay $B_s \to \pi^+\pi^-$. The naïve expectation for the corresponding branching ratio is $\mathcal{O}(10^{-8})$; a significant enhancement would signal that the "exchange" and "penguin annihilation" topologies cannot be neglected. Another interesting decay in this respect is $B_d \to K^+K^-$, for which already stronger experimental constraints exist [24].

3 Constraining the penguin parameters and the direct CP asymmetries

If we make use of (21) and apply the U-spin relation (16), the observable H allows us to determine the quantity

$$C \equiv \cos\theta\,\cos\gamma \qquad (26)$$

as a function of d:

$$C = \frac{a - d^2}{2\,b\,d}, \qquad (27)$$

where

$$a = \frac{1 - \epsilon^2 H}{H - 1} \qquad \text{and} \qquad b = \frac{1 + \epsilon H}{H - 1}. \qquad (28)$$

In [25], a similar function of strong and weak phases was considered for the $B_d \to \pi^\mp K^\pm$, $B^\pm \to \pi^\pm K$ system, and it was pointed out that this quantity plays an important role to derive interesting constraints. Since C is the product of two cosines, it has to lie between -1 and $+1$, thereby implying an allowed range for d. If we take into account (27) and (28), we obtain (for $H < 1/\epsilon^2$)

$$\frac{1 - \epsilon\sqrt{H}}{1 + \sqrt{H}} \leq d \leq \frac{1 + \epsilon\sqrt{H}}{|1 - \sqrt{H}|}. \qquad (29)$$

An alternative derivation of this range can be found in [7]. In the special case of $H = 1$, there is only a lower bound on d, which is given by $d_{\min} = (1 - \epsilon)/2$; for $H < 1$, C takes a minimal value that implies an allowed range for γ:

$$| \cos \gamma | \geq C_{\min} = \frac{\sqrt{(1 - \epsilon^2 H)(1 - H)}}{1 + \epsilon H} \approx \sqrt{1 - H}. \quad (30)$$

From a conceptual point of view, this bound on γ is completely analogous to the one derived in [25]. Unfortunately, it is only of academic interest in the present case, as (25) indicates $H > 1$, which we shall assume in the following discussion. So far, we have treated θ and γ as "unknown", free parameters. However, for a given value of γ, we have

$$-| \cos \gamma | \leq C \leq +| \cos \gamma |, \quad (31)$$

and obtain constraints on d that are stronger than (29):

$$d_{\min}^{\max} = \pm b | \cos \gamma | + \sqrt{a + b^2 \cos^2 \gamma}. \quad (32)$$

In Fig. 1, we show the dependence of C on d for the values of the observable H given in (25). Interestingly, the large values of H imply a rather restricted range for d. In particular, we get the lower bound $d \geq 0.2$. The "diamonds" in Fig. 1 represent the results obtained within the "QCD factorization" approach [20], representing the state-of-the-art technology in the calculation of the penguin parameter $d\,e^{i\theta}$:

$$d\,e^{i\theta}\big|_{\mathrm{QCD-fact}} = 0.09\,[0.18]\,e^{i\,193\,[187]^\circ}. \quad (33)$$

Here a certain formally power-suppressed contribution, which is "chirally enhanced" through the factor

$$r_\chi = \frac{2 M_\pi^2}{(m_u + m_d)\,m_b}, \quad (34)$$

has been neglected [included at leading order]. The "error bars" in Fig. 1 correspond to the presently allowed range for γ that is implied by the usual "indirect" fits of the unitarity triangle [26]:

$$36^\circ \leq \gamma \leq 97^\circ, \quad (35)$$

and the "diamonds" are evaluated with (33) for the preferred (central) value of $\gamma = 62^\circ$. The horizontal dotted lines in Fig. 1 represent $C = \pm \cos 36^\circ$. It is an interesting feature of the contours in the d–C plane that they allow in principle the determination of $\cos \gamma$ with the help (33), i.e. if d and θ are known. However, as can be seen in Fig. 1, the most recent CLEO data on $B_d \to \pi^+ \pi^-$ and $B_d \to \pi^\mp K^\pm$ are not in favour of an interpretation of the "QCD factorization" result (33) within the Standard Model; a solution could be obtained for $d \approx 0.2$ and $C \approx 1$. However, since (33) gives $\cos \theta \approx -1$, we would then conclude that $\cos \gamma \approx -1$, which would be in conflict with the Standard-Model range (35). Arguments for $\cos \gamma < 0$ using $B \to PP$, PV and VV decays were also given in [27].

Before we discuss the origin of a possible discrepancy of the "QCD factorization" results with the contours in the d–C plane, let us have a closer look at the impact of corrections to (16). To this end, we generalize this relation as follows:

$$d' = \xi\,d, \quad \theta' = \theta + \Delta\theta, \quad (36)$$

yielding

$$C \equiv \cos \theta\,\cos \gamma$$
$$= \left(\frac{1}{1 + u^2} \right)$$
$$\times \left[\frac{a - d^2}{2\,b\,d} \pm u \sqrt{(1 + u^2) \cos^2 \gamma - \left(\frac{a - d^2}{2\,b\,d} \right)^2} \right], (37)$$

where a and b correspond to the following generalization of (28):

$$a = \frac{1 - \epsilon^2 H}{\xi^2 H - 1}, \quad b = \frac{1 + \epsilon \xi H \cos \Delta\theta}{\xi^2 H - 1}, \quad (38)$$

and

$$u = \frac{\epsilon \xi H \sin \Delta\theta}{1 + \epsilon \xi H \cos \Delta\theta}. \quad (39)$$

Since the parameter u is doubly suppressed by ϵ and $\Delta\theta$, it is a small quantity. In the case of $\Delta\theta = 20^\circ$, $\xi = 1$ and $H = 7.4$, we have, for example, $u = 0.10$. In Fig. 2, we illustrate the impact of $\xi \neq 1$ and $\Delta\theta \neq 0$ on the contour in the d–C plane corresponding to $H = 7.4$. In contrast to (27), the general expression (37) depends also on the CKM angle γ for $\Delta\theta \neq 0$. However, since the major effect in Fig. 2 is due to possible corrections to $d' = d$, we shall assume $\theta' = \theta$ in the remainder of this paper. In this case, (37) takes the same form as (27).

Although it is too early to draw any definite conclusions, let us note that there would be basically two different explanations for a discrepancy of the "QCD factorization" results with the contours shown in Figs. 1 and 2: hadronic effects or physics beyond the Standard Model. Concerning the former case, the $\Lambda_{\mathrm{QCD}}/m_b$ terms and the "chirally enhanced" contributions may actually play an important role. Interestingly, the inclusion of the latter ones at leading order shifts the value of d in the right direction. In order to get the full picture, it would be an important task to analyse (20) and (36) in the "QCD factorization" approach. Using present data, it seems that the "QCD factorization" results (33) can only be accommodated – if at all possible – for values of γ sizeably larger than 90°, which would be in conflict with (35), and a possible sign for new physics. Since the parameter $d\,e^{i\theta}$ is governed by penguin topologies, i.e. by flavour-changing neutral-current (FCNC) processes, it may well be affected by physics beyond the Standard Model [28,29]. Moreover, it should be kept in mind that the unitarity of the CKM matrix has been used in the calculation of the contours shown in Figs. 1 and 2. Further studies and better data are needed to explore these exciting issues in more detail.
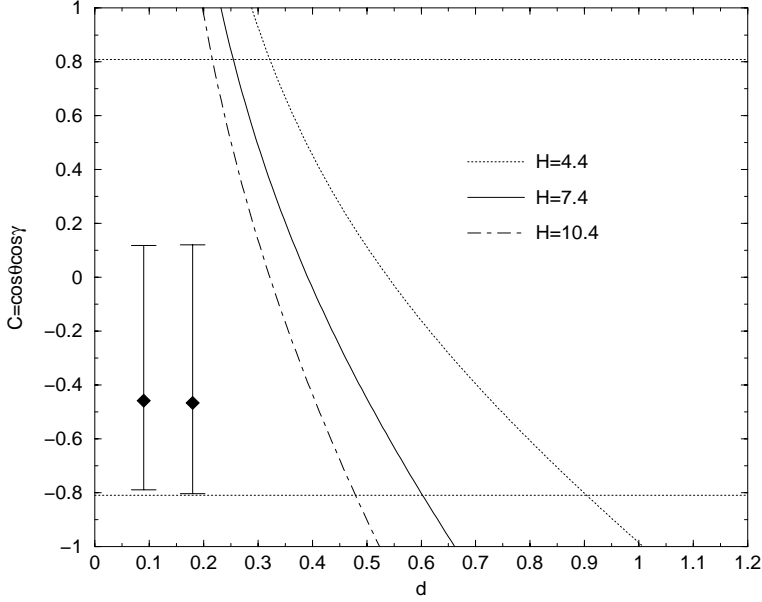
Fig. 1. The dependence of $C = \cos\theta\cos\gamma$ on the penguin parameter d for various values of the observable H. The "diamonds" with the error bars represent the results of the "QCD factorization" approach [20] for the presently allowed range of γ, as explained in the text. The horizontal dotted lines correspond to $C = \pm\cos 36°$
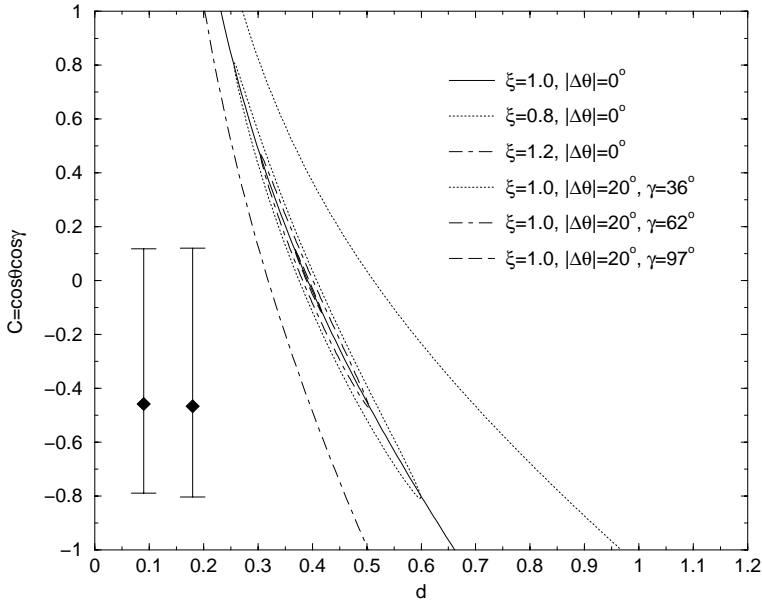

Fig. 2. The impact of corrections to (16), parametrized through $d' = \xi d$ and $\theta' = \theta + \Delta\theta$, on the contour in the d–C plane corresponding to $H = 7.4$

Let us now turn to the constraints on the direct CP asymmetries (see also [7,25]). Before turning to the general case, it is instructive to consider $\gamma = 90°$. In this case, we obtain

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-)\big|_{\gamma=90°} = -\left[\frac{2\,d\sin\theta}{1+d^2}\right],$$

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to K^+K^-)\big|_{\gamma=90°} = +\left[\frac{2\,\epsilon\,d'\sin\theta'}{\epsilon^2+d'^2}\right], \quad (40)$$

and

$$H\big|_{\gamma=90°} = \frac{1+d^2}{\epsilon^2+d'^2}. \qquad (41)$$

The CP asymmetries given in (40) take their extremal values for $\theta = \theta' = \pm 90°$, and (41) allows us to determine d:

$$d\big|_{\gamma=90°} = \sqrt{\frac{1-\epsilon^2 H}{\xi^2 H - 1}}, \qquad (42)$$

where we have also used $d' = \xi d$. Consequently, we obtain

$$\left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-)\right|_{\gamma=90°}^{\max} = 2\sqrt{\frac{(1-\epsilon^2 H)(\xi^2 H - 1)}{(\xi^2-\epsilon^2)^2 H^2}}$$

$$\approx \frac{2}{\xi\sqrt{H}} \qquad (43)$$

and

$$\left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to K^+K^-)\right|_{\gamma=90°}^{\max} = 2\,\epsilon\,\xi\sqrt{\frac{(1-\epsilon^2 H)(\xi^2 H - 1)}{(\xi^2-\epsilon^2)^2}}$$

$$\approx 2\,\epsilon\,\sqrt{H}. \qquad (44)$$

Let us emphasize that (44) is essentially *unaffected* by any corrections to the U-spin relation (16) for $H = \mathcal{O}(10)$; its theoretical accuracy is practically only limited by (20), which enters in the determination of H through (17).

In the general case $\gamma \neq 90°$, we employ (27) to eliminate the CP-conserving strong phase θ in (10). Following these lines, we obtain $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-)$ as a function of d for a given value of γ. If we keep γ fixed, and vary d within the allowed range corresponding to (32), we find that $|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-)|$ takes the following maximal value:

$$
\begin{aligned}
&\left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-)\right|_{\max} \\
&= 2|\sin\gamma|\sqrt{\frac{a + b^2\cos^2\gamma}{(1+a)^2 - 4(a-b)(1+b)\cos^2\gamma}}, \quad (45)
\end{aligned}
$$

where a and b are given in (38) for $\Delta\theta = 0$ (see the comment after (39)). In the case of $B_s \to K^+K^-$, we obtain

$$
\begin{aligned}
&\left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^\mp K^\pm)\right|_{\max} \approx \left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to K^+K^-)\right|_{\max} \\
&= 2\,\epsilon\,\xi\,H\,|\sin\gamma|\sqrt{\frac{a + b^2\cos^2\gamma}{(1+a)^2 - 4(a-b)(1+b)\cos^2\gamma}}. \quad (46)
\end{aligned}
$$

For $\gamma = 90°$, these expressions reduce to (43) and (44), respectively. In Fig. 3, we show the dependence of (45) and (46) on γ for the values of H given in (25). The shaded regions correspond to a variation of the parameter $\xi \equiv d'/d$ within the interval $[0.8, 1.2]$ for $H = 7.4$. In contrast to (45), (46) is essentially unaffected by a variation of ξ, as we have already noted above. The range for H given in (25) disfavours large direct CP violation in $B_s \to K^+K^-$ and $B_d \to \pi^\mp K^\pm$ (see also [29]), which is also consistent with the 90% C.L. interval of $-0.22 \leq \mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^\mp K^\pm) \leq +0.30$ reported recently by the CLEO collaboration [14]. On the other hand, there is a lot of space for large direct CP violation in $B_d \to \pi^+\pi^-$. As can be seen in Fig. 3, a measurement of non-vanishing CP asymmetries $|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}|_{\exp}$ would allow us to exclude immediately a certain range of γ around $0°$ and $180°$, as values of γ corresponding to $|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}|_{\exp} > |\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}|_{\max}$ are excluded. However, in order to constrain this CKM angle, the mixing-induced CP asymmetry $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)$ appears to be more powerful.

Before we turn to these bounds in the following section, let us note that the observables of the decay $B_d \to \pi^+\pi^-$ were combined with the CP-averaged $B_d \to \pi^\mp K^\pm$ and $B_s \to K^+K^-$ branching ratios in [6] and [7], respectively, to derive constraints on the penguin effects in the extraction of the CKM angle α. In the present paper, we combine the experimental information provided by these modes in a different way, which appears more favourable to us. In particular, we use the mixing-induced CP asymmetry of the "gold-plated" mode $B_d \to J/\psi K_{\mathrm{S}}$ as an additional input [8], and derive bounds on the CKM angle γ. The utility of $B_d \to \pi^\mp K^\pm$ decays to control the penguin effects on CP violation in $B_d \to \pi^+\pi^-$ was also emphasized in [30].

4 Constraining the CKM angle γ

In the following discussion, we assume that $\phi_d = 2\beta$ has been measured at the B-factories through (12), which is one of the major goals of these experiments. The presently allowed range for β that is implied by the usual "indirect" fits of the unitarity triangle is given as follows [26]:

$$16° \leq \beta \leq 35°, \tag{47}$$

with a preferred (central) value of $\beta = 25°$, which is also consistent with the present experimental result $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{S}}) = -\sin(2\beta) = -0.79^{+0.44}_{-0.41}$ of the CDF collaboration [31]. A measurement of this mixing-induced CP asymmetry allows us to determine only $\sin\phi_d$, i.e. to fix ϕ_d up to a twofold ambiguity. Several strategies were proposed in the literature to resolve this ambiguity [32]. In the B-factory era, an experimental uncertainty of $\Delta \sin\phi_d\,|_{\exp} = 0.05$ seems to be achievable after a few years of taking data, which corresponds to an uncertainty of $\Delta\phi_d = \pm 5°$ for the central value of $\phi_d = 50°$.

If we assume, for a moment, that there are no penguin effects present in $B_d \to \pi^+\pi^-$, i.e. $d = 0$, we would simply have

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)\big|_{d=0} = \sin(\phi_d + 2\gamma), \tag{48}$$

as can be seen in (11). Since the unitarity of the CKM matrix implies $\phi_d + 2\gamma = -2\alpha$, this CP asymmetry is usually written as $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)|_{d=0} = -\sin(2\alpha)$, and would allow a direct measuerment of α. However, (48) is the "generic" interpretation of this CP asymmetry, allowing us to determine γ, if ϕ_d is fixed through $B_d \to J/\psi K_{\mathrm{S}}$. In the case of large penguin contributions, this interpretation of $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)$ actually appears to be more favourable than the usual one in terms of α, which was employed, for example, in [6,7]. Since we definitely have to worry about penguin effects in $B_d \to \pi^+\pi^-$, as we have pointed out in the previous section, we shall use the corresponding mixing-induced CP asymmetry to contrain the CKM angle γ in this section.

Concerning the search for new physics, γ is actually the interesting aspect of the mixing-induced $B_d \to \pi^+\pi^-$ CP asymmetry. If ϕ_d is affected by new physics, these effects could be seen, for example, by comparing the $B_d \to J/\psi K_{\mathrm{S}}$ results with the "indirect" range (47). Since this channel is governed by $\bar{b} \to \bar{c}c\bar{s}$ "tree" processes, its decay amplitude is not expected to be affected significantly by new-physics effects, and allows the determination of ϕ_d even in the presence of physics beyond the Standard Model. In order to search for indications of new physics, the values of γ implied by the CP-violating effects in $B_d \to \pi^+\pi^-$ could be compared with the "indirect" range arising from the usual fits of the unitarity triangle, or with theoretically clean extractions from pure "tree" decays, such as $B_d \to D^{*\pm}\pi^\mp$ or $B \to DK$ (see also the brief discussion of new-physics effects in Sect. 3).

If we look at the expressions (10) and (11) for the direct and mixing-induced CP asymmetries of the decay $B_d \to \pi^+\pi^-$, we observe that the CP-conserving strong

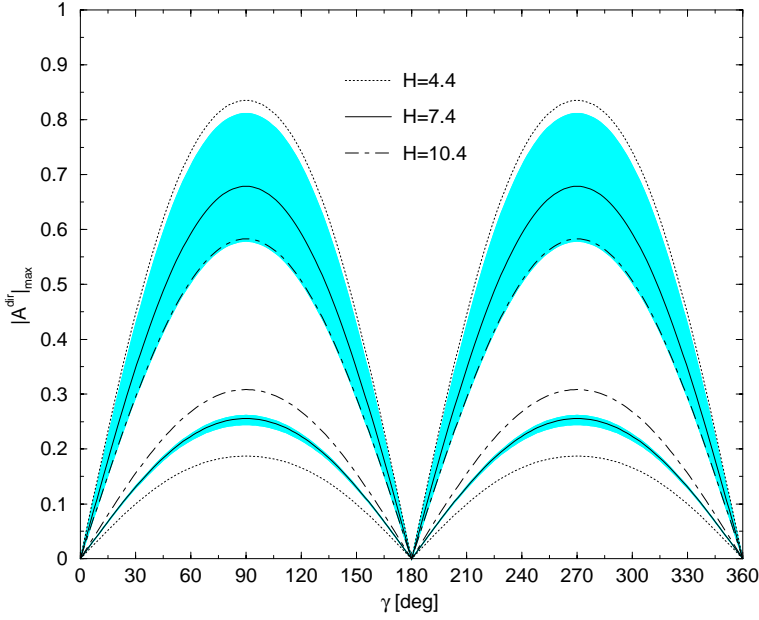**Fig. 3.** The dependence of the maximally allowed direct CP-violating asymmetries $\left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-)\right|_{\max}$ (thin lines) and $\left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to K^+K^-)\right|_{\max} \approx \left|\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^{\mp}K^{\pm})\right|_{\max}$ (thick lines) on the CKM angle γ for various values of the observable H. The shaded regions correspond to a variation of ξ within the interval $[0.8, 1.2]$ for $H = 7.4$
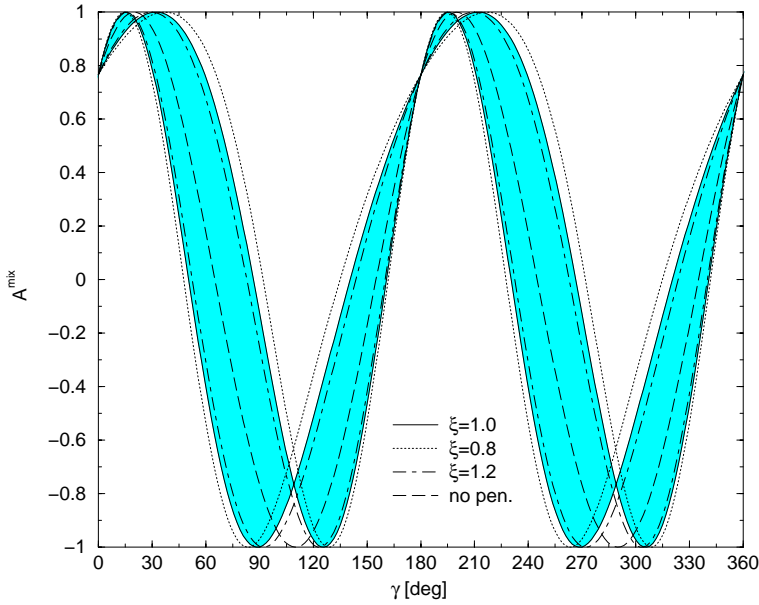

Fig. 4. The dependence of the allowed range for $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)$ on the CKM angle γ for $H = 7.4$ and $\phi_d = 50°$

phase θ enters only in the form of $\cos\theta$ in the latter case. Consequently, using $\cos\theta = C/\cos\gamma$ and (27), we obtain

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-) = \tag{49}$$
$$\frac{[\, b\,\sin(\phi_d+2\gamma)\cos\gamma - a\,\sin(\phi_d+\gamma)] + [\sin(\phi_d+\gamma) + b\,\sin\phi_d\,\cos\gamma]d^2}{[(b-a)+(1+b)\,d^2]\cos\gamma},$$

where a and b are given in (38) for $\Delta\theta = 0$, i.e. the small corrections due to $\Delta\theta \neq 0$ have been neglected for simplicity (see the comment after (39)). Since (49) is a monotonic function of the variable d^2, it takes its extremal values for the minimal and maximal values of d given in (32); inserting them into (49) yields

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)\big|_{\mathrm{extr.}} = \tag{50}$$
$$\frac{\sin(\phi_d+2\gamma) + a\,\sin\phi_d + 2\,w_{\pm}\,[\sin(\phi_d+\gamma) + b\,\cos\gamma\,\sin\phi_d]}{1 + a + 2\,w_{\pm}(1+b)\cos\gamma},$$

where

$$w_{\pm} = b\cos\gamma \pm \sqrt{a + b^2\cos^2\gamma}. \tag{51}$$

In Fig. 4, we illustrate the resulting allowed range for $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)$ in the case of $H = 7.4$ and $\phi_d = 50°$ (shaded region). The impact of a deviation of the parameter ξ from 1 is illustrated by the dotted and dot-dashed lines, which correspond to $\xi = 0.8$ and 1.2, respectively. For a given value of γ, the allowed range for the mixing-induced $B_d \to \pi^+\pi^-$ CP asymmetry is usually very large. However, a measured value of $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-)$ would, on the other hand, imply a rather restricted range for γ. If we assume, for example, that $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-) = 0.4$ has been measured, and take into account that the experimental value of ε_K implies $\gamma \in [0°, 180°]$, we would conclude that $41° \leq \gamma \leq 74°$ or $158° \leq \gamma \leq 170°$. Al-

lowing $\xi \in [0.8, 1.2]$, i.e. symmetry-breaking corrections of 20%, we would obtain the slightly modified ranges $39° \leq \gamma \leq 80° \vee 155° \leq \gamma \leq 170°$ and $43° \leq \gamma \leq 71° \vee 160° \leq \gamma \leq 170°$ for $\xi = 0.8$ and 1.2, respectively. Since the allowed region for d is enlarged (reduced) for smaller (larger) values of H, the bounds on γ become weaker (stronger) in this case.

Let us finally note that if in addition to a measurement of H and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+ \pi^-)$ direct CP violation in $B_d \to \pi^+ \pi^-$ or $B_d \to \pi^\mp K^\pm$ is observed, we have three independent observables at our disposal, which depend on γ, d and θ. Consequently, we are then not only in a position to constrain these "unknown" parameters, but also to *determine* them [8]. Moreover, the normalization $|\mathcal{C}|$ of the $B_d \to \pi^+ \pi^-$ decay amplitude (see (7)) can be extracted from the corresponding CP-averaged branching ratio, and can be compared with theoretical predictions. The $B_d \to \pi^\mp K^\pm$ decays offer also alternative strategies to determine γ and certain hadronic quantities, if these transitions are combined with other $B \to \pi K$ modes [33].

5 Conclusions and outlook

The decays $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$ provide interesting strategies to extract the CKM angle γ and hadronic penguin parameters at "second-generation" B-physics experiments of the LHC era. In this paper, we have considered a variant of this approach for the "first-generation" B-factories, where the $B_s \to K^+ K^-$ decays are replaced by $B_d \to \pi^\mp K^\pm$ modes.

We have pointed out that the CP-averaged $B_d \to \pi^+ \pi^-$ and $B_d \to \pi^\mp K^\pm$ branching ratios allow us to fix contours in the d–$[\cos \theta \cos \gamma]$ plane, which can be compared with theoretical results for the $B_d \to \pi^+ \pi^-$ "penguin parameter" $d\,e^{i\theta}$, for example with those of the "QCD factorization" approach. Although it is too early to draw any definite conclusions, it is interesting to note that the most recent CLEO data are not in favour of an interpretation of the "QCD factorization" results within the Standard Model. This feature may be due to hadronic effects or new physics. Further theoretical studies and better experimental data are required to investigate these exciting issues in more detail.

Another interesting aspect of the recent CLEO results for the CP-averaged $B_d \to \pi^+ \pi^-$ and $B_d \to \pi^\mp K^\pm$ branching ratios is that they imply upper bounds on the corresponding direct CP asymmetries, which are given by $|\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+ \pi^-)|_{\rm max} \lesssim 0.8$ and $|\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)|_{\rm max} \approx |\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+ K^-)|_{\rm max} \lesssim 0.3$. The latter bound is remarkably stable under U-spin-breaking corrections – in contrast to the former one – and may also play an important role to search for new physics.

If in addition to the CP-averaged $B_d \to \pi^+ \pi^-$ and $B_d \to \pi^\mp K^\pm$ branching ratios mixing-induced CP violation in the former decay is measured, and the B_d^0–$\overline{B_d^0}$ mixing phase is fixed through $B_d \to J/\psi K_{\rm S}$, interesting constraints on γ can be obtained. A further step in this programme would be the observation of direct CP violation in $B_d \to \pi^+ \pi^-$ or $B_d \to \pi^\mp K^\pm$, which would allow a

determination of γ, $d\,e^{i\theta}$ and $|\mathcal{C}|$. In this way, two of the major goals of the B-factories – time-dependent analyses of the benchmark modes $B_d \to J/\psi K_{\rm S}$ and $B_d \to \pi^+ \pi^-$ – can be combined with each other to probe the CKM angle γ and to obtain valuable insights into the world of penguins.

Another important step would be a measurement of the CP-averaged $B_s \to K^+ K^-$ branching ratio, which may be possible at HERA-B and Run II of the Tevatron. Using this observable, a certain dynamical assumption concerning "exchange" and "penguin annihilation" topologies can be avoided, which has to be made in the case of $B_d \to \pi^\mp K^\pm$. The theoretical accuracy would then only be limited by U-spin-breaking effects and would not be affected by any final-state-interaction processes. The final goal is a measuerment of the CP-violating observables of $B_s \to K^+ K^-$, which should be possible at LHCb and BTeV. At these experiments, the physics potential of $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$ can be fully exploited, and in addition to an extraction of γ at the level of a few degrees, also interesting consistency checks of the basic U-spin relations can be performed.

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